the Reynolds and Nusselt numbers. The streamwise array spacing-to-channel height and clearance height-to-element length ratios, LS/H and $(H-B)/L$, were identified as important geometric parameters. A correlation for experiments performed in air, water and FC-77 was developed [equation (3)]. All the liquid data were very well correlated by the proposed correlation, but larger deviations with the air data prompted the development of an improved correlation for air data alone [equation (4)]. Both correlations compared favorably with predictive formulae in the literature.

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Heat transport along an oscillating flat plate in the presence of a transverse magnetic field

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1. INTRODUCTION

In recent years, several investigations were carried out to study the characteristics of a novel heat transport mode in which heat is transported from a hot to a cold reservoir by means of sinusoidal oscillations of a viscous fluid contained within open-ended tubes connecting the reservoirs. It has been experimentally [1] and analytically [2] confirmed that such periodic longitudinal oscillations result in a very significant enhancement in axial transport capability of the fluid.

A significant aspect of this thermal transport technique is that it involves no net convective mass transfer. The application of this method to several engineering problems is cited in ref. [3].

Very recently Kurzweg [4] showed that the same enhanced heat transfer process can occur in the classical Stokes problem of sinusoidally oscillating flat plate immersed within a viscous fluid of infinite extent when a constant temperature gradient is superimposed on the fluid parallel to the direction of oscillation of the plate.

However, so far no attempt has been made to study the effect of a magnetic field over such heat transport in electrically conducting fluid flows, despite the fact that in the devices using this thermal pumping, liquid metals such as mercury, liquid lithium or sodium are preferable [5]. Such an investigation will not only be useful in the design, control and improvement of liquid metal heat exchangers [6] but also may throw some light on the possible coupling of solar thermal system with liquid metal MHD heat exchangers [7].

NOMENCLATURE

A_0	amplitude of oscillation [m]
B_0	magnetic induction [webers m^{-2}]
$\mathbf{B}_0$	magnetic induction vector [webers m^{-2}]
c	specific heat [J kg^{-1} K^{-1}]
$\mathbf{E}$	electric field vector [volts m^{-1}]
$\mathbf{j}$	current density vector [amp m^{-2}]
k	thermal diffusivity [m^2 s^{-1}]
M	Hartmann number, $B_0\,\delta\sqrt{\sigma/(\rho v)}$
Pe	Peclect number, $(\omega A_0\delta)/k$
Pr	Prandtl number, v/k
Q	total time-averaged heat flux [W]
Q_1	non-dimensional total time-averaged heat flux
T	temperature [K]
t	time [s]
U	velocity [m s^{-1}]
W	width of the plate [m]
x, y	space variables [m].

Greek symbols

γ	constant temperature gradient [K m^{-1}]
δ	Stokes layer thickness [m]
η	non-dimensional transverse space variable, y/δ
v	kinematic coefficient of viscosity [m^2 s^{-1}]
ρ	density [kg m^{-3}]
σ	electrical conductivity [Coulomb m^{-3}]
τ	non-dimensional time, ωt
ϕ	thermal flux [W m^{-2}]
$\bar{\phi}$	time-averaged thermal flux [W m^{-2}]
ω	angular oscillation frequency [rad s^{-1}].

The present study is an attempt to investigate the effect of transverse magnetic field over the heat transfer introduced by Kurzweg [4].

2. FORMULATION AND SOLUTION

Consider a very thin thermally and electrically insulated infinite flat plate oscillating sinusoidally within an unbounded viscous, incompressible, electrically conducting fluid of infinite extent.

The amplitude of oscillation is A_0 and the movement of the plate is parallel to the fluid solid interface, the x-direction. A constant temperature gradient $\gamma = \partial T/\partial x$ is superimposed on the fluid in the oscillation direction. A transverse magnetic field $\mathbf{B}_0 = B_0\hat{j}$ is applied in the y-direction. In order that return paths for the current and magnetic fields be clearly specified, the flow over the flat plate in a uniform magnetic field should be regarded as the limit of a three-dimensional problem such as the flow over the cylinder, when the various radii become very large, with the inner, outer poles and magnetic yoke as cited in Shercliff [8] and hence experiments are to be performed using infinite cylinders in the place of infinite flat plates

The governing equation of the problem is

$$\partial U/\partial\tau = (1/2)[\partial^2 U/\partial\eta^2 - M^2 U], \tag{1}$$

where $\eta = y/\delta$, $\tau = \omega t$, $M = B_0\delta\sqrt{\sigma/(\rho v)}$, the Hartmann number, with ω the angular oscillation frequency, ρ is the density, v the kinematic coefficient of viscosity, σ the electrical conductivity of the fluid, $\delta = \sqrt{(2v/\omega)}$, the Stokes layer thickness and U the laminar velocity field component in the x-direction. Here, equation (1) is derived following the lines of Stokes problem for the MHD case. Further, in deriving this equation, the induced magnetic field is considered to be negligible in comparison to the applied field and hence, is neglected [6]. Since $\mathbf{B}_0$ is uniform and the induced magnetic field is neglected, curl $\mathbf{E} = 0$. Also, div $\mathbf{E} = 0$ in the absence of surface charge density. Hence $\mathbf{E} = 0$. Further, in this case, it is seen that div $\mathbf{j} = 0$, where $\mathbf{E}$ and $\mathbf{j}$ are the electric field and current density vectors.

The initial and boundary conditions are:

$$U = 0 \quad \text{for all } \eta \geq 0 \quad \text{and} \quad \tau = 0$$

$$U = A_0\omega\cos\tau \quad \text{for} \quad \eta = 0 \quad \text{and all} \quad \tau > 0$$

and

$$U = 0 \quad \text{for} \quad \eta \to \infty. \tag{2}$$

Assuming $U(\eta,\tau) = Re\{\exp(i\tau)f(\eta)\}$ the velocity field is obtained as

$$U(\eta,\tau) = A_0\omega\exp(-A\eta)\cos(\tau - B\eta), \tag{3}$$

where

$$A = \sqrt{\frac{M^2+M^4+4}{2}}, \quad B = \sqrt{\frac{-M^2+M^4+4}{2}}.$$

The energy equation reduces to

$$[\omega(\partial T/\partial\tau) + U\gamma] = (k/\delta^2)(\partial^2 T/\partial\eta^2), \tag{4}$$

neglecting viscous and joule dissipation, where T is the temperature and k is the thermal diffusivity of the fluid.

The corresponding boundary conditions are:

$$\partial T/\partial\eta = 0 \quad \text{at} \quad \eta = 0 \quad \text{and when} \quad \eta \to \infty. \tag{5}$$

Assuming a temperature distribution in the form [9]

$$T(x,\eta,\tau) = \gamma Re\{x + \delta g(\eta)\exp(i\tau)\}, \tag{6}$$

we obtain

$$g(\eta) = -Pe(C+iD)\exp[-(1+i)\sqrt{Pr}\eta]/[2\sqrt{Pr}E]$$
$$+ Pe[M^2 - 2i(1-Pr)]\exp[-(A+iB)\eta]/E, \tag{7}$$

where $Pr = v/k$ is the Prandtl number, $Pe = \omega A_0\delta/k$ is the Peclect number, $C = (A+B)M^2 + 2(B-A)(1-Pr)$, $D = (B-A)M^2 - 2(1-Pr)(A+B)$ and $E = M^4 + 4(1-Pr)^2$.

3. HEAT FLUX AND THERMAL FLUX BOUNDARY LAYER

Following the lines of Kurzweg [4], the net heat flow produced by the interaction of longitudinal convection and transverse conduction across the Stokes layer, neglecting the very small contribution due to conduction in the x-direction can be mathematically expressed as longitudinal thermal flux at $x = 0$ as

$$\phi = \rho c U T(0,\eta,\tau).$$

Upon time averaging ϕ over one period of the sinusoidal oscillation, we obtain

$$\hat{\phi} = (1/2\pi)\int_0^{2\pi} \phi \, d\tau.$$

One can integrate $\hat{\phi}$ over the entire range of η to obtain the total time averaged heat transport for a W wide plate as

$$Q = W\delta \int_0^{\infty} \hat{\phi} \, d\eta = -(W\delta PcA_0^2\gamma\omega)Q_1,$$

where Q_1 is the non-dimensional total time averaged heat transported. It is evident that

$$Q_1 = H\sqrt{Pr}/(2E),$$

where

$$H = [C(A+\sqrt{Pr})-D(B-\sqrt{Pr})]/[(A+\sqrt{Pr})^2 + (B-\sqrt{Pr})^2] - M^2\sqrt{Pr}/A.$$

Finally the fraction $F(\eta)$, which is the ratio of the time averaged flux $\hat{\phi}$ passing through thickness η of fluid near the plate of width W to Q, is calculated. That is,

$$F(\eta) = \left[W\delta \int_0^{\eta} \hat{\phi} \, d\eta\right] \Big/ Q$$

$$= 1.0 + (1/H)[\psi_1(\eta)\psi_2(\eta) + \psi_3(\eta)], \quad (8)$$

where

$$\psi_1(\eta) = \{\exp[-(A+\sqrt{Pr})\eta]\}/[(A+\sqrt{Pr})^2 + (B-\sqrt{Pr})^2] \quad (9)$$

$$\psi_2(\eta) = [CB+DA+\sqrt{Pr}(D-C)]\sin[(B-\sqrt{Pr})\eta] + [DB-AC-\sqrt{Pr}(C+D)]\cos[(B-\sqrt{Pr})\eta] \quad (10)$$

and

$$\psi_3(\eta) = M^2\sqrt{Pr}\exp(-2A\eta)/A. \quad (11)$$

Defining thermal flux boundary layer as the thin layer near the plate within which 99% of the total flux is transported, the thermal flux boundary layer thickness is the value of η for which $F(\eta)$ is 0.99.

4. DISCUSSION AND CONCLUSION

It is inferred from equation (3) that the higher the value of Hartmann number, the sooner the velocity becomes zero in the η direction, which confirms the creation of Hartmann boundary layer and this fact is true for all values of τ. Thus, the effect of the magnetic field is to stabilize the system and decelerate the flow.

Equation (8) reveals the fact that apart from modifying the thermal boundary layer, the effect of the magnetic field is to introduce a new thermomagnetic boundary layer near the plate, which is clearly inferred from equation (11).

It is seen from Table 1 that the effect of the magnetic field is to decrease the thermal flux boundary layer thickness when $M > 1.0$, for any Prandtl number. Similarly, for $M > 1.0$, the effect of Prandtl number is to decrease the thickness of this layer. The maximum value of this boundary layer thickness is not affected by the presence of the magnetic field and it remains 3.754 times the Stokes boundary layer.

Fig. 1 clearly discloses the fact that the magnetic field effect is to decrease the thermal flux transported.

To conclude, the effect of the magnetic field over the heat transfer is not only quantitative but also qualitative. It reduces the thermal flux by decelerating the flow. It not only decreases the thermal flux boundary layer thickness but also creates a thermomagnetic boundary layer near the plate. In the non-magnetic case the results are identical to those of ref. [4].

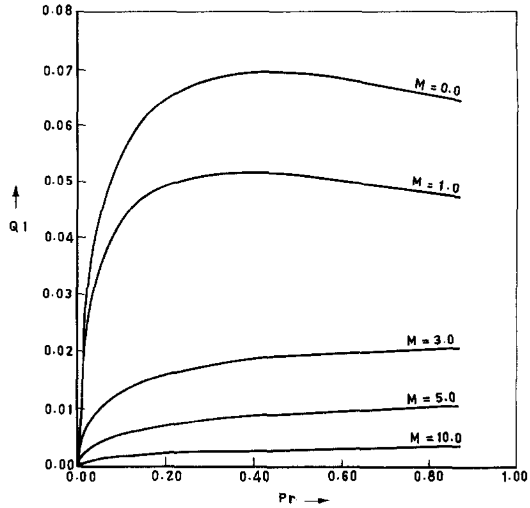

Fig. 1. Effect of magnetic field over time-averaged heat flux.

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Table 1. Thermal flux boundary layer thickness for various Pr and M

	M					
Pr	0.0	0.5	0.9	3.0	5.0	10.0
0.0	2.2870	2.2870	2.2870	1.6310	0.9519	0.4649
0.01	2.6049	2.6049	2.6049	1.5760	0.9239	0.4569
0.044	2.9859	2.9859	2.9859	1.5050	0.8909	0.4479
0.08	3.2869	3.2869	3.2869	1.4400	0.8619	0.4409
0.10	3.3369	3.3369	3.3369	1.4270	0.8569	0.4389
0.36	3.7540	3.7540	3.7540	1.1910	0.7579	0.4139
0.7	3.5689	3.5689	3.5689	0.9809	0.6729	0.3929
1.0	3.3199	3.3199	3.3199	0.8459	0.6139	0.3769
2.3	2.4660	2.4660	2.4660	0.5529	0.4539	0.3279
10.0	1.0549	1.0549	1.0549	0.2400	0.2200	0.2010
100.0	0.2610	0.2610	0.2610	0.0659	0.0639	0.0629

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